EFFECTS OF VARIOUS PARAMETERS ON LATERAL DISPLACEMENT ESTIMATION IN ULTRASOUND ELASTOGRAPHY

JIANWEN LUO* and ELISA E. KONOFAGOU*†

*Ultrasound and Elasticity Imaging Laboratory, Department of Biomedical Engineering; and †Department of Radiology, Columbia University, New York, NY, USA

(Received 8 September 2008, revised 18 February 2009, in final form 3 March 2009)

Abstract—Complementary to axial, lateral displacement and strain can provide important information on the biological soft tissues. In this paper, the effects of key parameters (i.e., lateral displacement, pitch, beamwidth, beam overlap and interpolation) on lateral displacement estimation were investigated, in simulations and homogeneous phantom experiments, using lateral rigid motion only to study its fundamentals separately from the effects of axial motion and 2-D deformation on lateral displacement estimation. The performance of the lateral motion estimator was evaluated by measuring its associated bias, jitter and correlation coefficient. Simulation results showed that the bias and jitter of the lateral displacement estimation and correlation coefficient of RF signals undergo periodic variations depending on the lateral displacement, with a period equal to the pitch. The performance of the lateral estimation was improved when a smaller pitch or a larger beamwidth, was used. The effect of the pitch on the lateral estimation on lateral displacement estimation was found to be greater than the beamwidth effect. Therefore, a smaller pitch is preferred when the beam overlap remains the same. The use of cubic spline, instead of linear interpolation, increases the correlation coefficient, and decreases the jitter, with the trade-off of increased bias. The results of the phantom experiments were shown in good agreement with the simulation findings, including the periodic variation of the performance with lateral displacement and effects of pitch, beamwidth and interpolation method on lateral displacement estimation. In conclusion, smaller pitch, wider beamwidth and spline interpolation were shown to be key in reducing the jitter error in the lateral displacement estimation. (E-mail: ek2191@columbia.edu) © 2009 World Federation for Ultrasound in Medicine & Biology.

Key Words: Beam overlap, Beamwidth, Bias, Correlation Coefficient, Cross-correlation, Displacement, Elastography, Estimation, Imaging, Interpolation, Jitter, Lateral Displacement, Motion, Performance, Pitch, Radiofrequency (RF) signal, Spline, Strain.

INTRODUCTION

Elastography

Elastography has been developed into an effective imaging method of estimating the local elastic properties of biological tissues (Ophir et al. 1991). Typically, an external, static or quasi-static, compression is applied on the tissue. The resulting displacement distribution within the tissue is estimated using cross-correlation analysis on the acquired pre- and postcompression radiofrequency (RF) signals. The strain distribution is then calculated as the spatial gradient of the displacement. By solving the inverse problem, the shear modulus, or Young’s modulus, can be reconstructed from the estimated displacement and strain images, i.e., elastograms (Skovoroda et al. 1995; Kallel and Bertrand 1996).

Several different elastographic methods and applications have been developed over the past decades. For example, elastography has been applied successfully in the diagnosis of breast lesions and is currently used clinically (Hiltawsky et al. 2001; Itoh et al. 2006). By using the inherent contraction and relaxation of the myocardium as the stimulus, myocardial elastography has further been developed for noninvasively imaging regional myocardial function (Konofagou et al. 2002; Lee et al. 2007; Luo et al. 2007).

Lateral motion estimation

Typically, only the axial (i.e., along the ultrasound beam) displacement and strain are estimated in elastography because compression is applied in the axial direction in which the ultrasound RF signals are also acquired.
However, most biological tissues are nearly incompressible (Fung 1993), i.e., the axial compression leads to expansion in the lateral (i.e., perpendicular to ultrasound beams in the image plane) and elevational (i.e., perpendicular to the image plane) directions. Estimation of the lateral displacement and strain may provide important additional information on the tissue mechanical properties. Furthermore, in cardiac applications, the heart undergoes complex motion and deformation in the 3-D space. Estimation and interpretation of only the axial information may also result in angle-dependent artifacts (Zervantonakis et al. 2007).

Trahey et al. (1987) developed a 2-D velocity estimation method for blood flow imaging using 2-D kernel in B-mode images. In addition to the axial displacements and strains, elastography has been shown to be capable of obtaining lateral displacements and strains. Konofagou and Ophir (1998) proposed a lateral displacement estimation method by using the RF signal interpolation in the lateral direction so as to get subpitch lateral motion estimate. The correlation coefficient between a 1-D RF signal kernel in the reference frame and a 1-D kernel in the post-interpolation comparison frame was calculated within a 2-D search range. The peak location of the correlation coefficient provided both axial and lateral motion. The information on axial strain and lateral displacement was further used to recorrelate the RF signals and compensate for their decorrelation noise on the lateral and axial displacement estimation, respectively (Konofagou and Ophir 1998). Therefore, an iterative and recorrelation strategy was proposed to obtain more accurate axial and lateral displacement estimates as well as corresponding images. In Lee et al. (2007), an alternative recorrelation technique was implemented by shifting RF signal segments in the comparison frame according to the estimated axial displacement, before lateral displacement estimation. After recorrelation, the effects of axial motion and deformation on the image quality were effectively compensated.

The lateral displacement technique can then yield lateral strains shear strains and the Poisson’s ratio have been shown capable of being estimated (Konofagou and Ophir 1998; Konofagou et al. 2000a). Shear strains help distinguish different tumors based on their mobility (Konofagou et al. 2000a). Poisson’s ratio elastography and poroelastography have also been developed to estimate the Poisson’s ratio, an important mechanical parameter of tissues, and to describe spatial and temporal behavior of poroelastic materials, respectively (Konofagou et al. 2001; Righetti et al. 2004, 2005). In cardiac applications, the axial/lateral estimation of the myocardial strain were used to calculate the angle-independent radial, circumferential and principal strains in vivo (Lee et al. 2007; Zervantonakis et al. 2007).

**Performance evaluation**

To improve the performance of elastography, the effects of different parameters on the displacement and strain estimation need to be studied. Previously reported efforts have concentrated on the performance analysis of the axial displacement and strain estimation using different parameters (Walker and Trahey 1994, 1995; Bilgen and Insana 1997a, 1997b; Varghese and Ophir 1997b). However, there are only a few fundamental studies on the performance of the lateral displacement and strain estimation. In particular, the effects of lateral displacement, pitch, beamwidth and beam overlap on the lateral displacement and strain estimation have not been thoroughly studied.

Walker and Trahey (1994) derived analytical expressions predicting the magnitude of jitter for 1-D and 2-D time-delay estimation using cross-correlation. In that paper, a 1-D lateral kernel and a 2-D kernel were considered in the 1-D and 2-D problems, respectively. In this paper, the lateral displacement estimation method uses a 1-D axial kernel (Konofagou and Ophir 1998). In Walker and Trahey (1994), the RF signal interpolation in the lateral direction (Konofagou and Ophir 1998) had not been taken into account. Kallel and Ophir (1997) studied the effects of 3-D tissue motion on the axial strain estimation. Konofagou et al. (2000b) investigated the theoretical bound on the estimation of lateral displacement and strain. The theoretical effects of pitch, beamwidth, lateral strain and lateral interpolation were analyzed within the framework of the strain filter (Konofagou et al. 2000b). The theoretical analysis in those two studies (Kallel and Ophir 1997; Konofagou et al. 2000b) was based on the correlation coefficient between the pre- and postdeformation RF signals acquired at the same beam location (e.g., at beam n in both pre- and postdeformation frames). The correlation coefficient between consecutive RF signals at the same beam location could also be found in Viola and Walker (2002). However, RF signals obtained at different beam locations as a result of the lateral search and lateral interpolation (e.g., at beam n and n + Δn in the pre- and postdeformation frames, respectively, where Δn is the sub-beam lateral displacement) had not been taken into account in those studies (Kallel and Ophir 1997; Konofagou et al. 2000b; Viola and Walker 2002). As will be shown in this paper, the correlation coefficient, as well as the bias and jitter error of the lateral displacement estimation as a function of the magnitude of the lateral displacement, undergo periodic variations. This was not entirely represented by the previously developed theoretical analysis (Konofagou et al. 2000b). The effects of different interpolation methods on the lateral displacement estimation will also be studied in this paper.

The lateral motion leads to a decorrelation increase between the pre- and postdeformation RF signals. Kallel
and Ophir (1997) assumed that the point-spread function (PSF) components were separable and found that the effects of lateral displacement and axial strain on the axial strain estimation were also separable. Therefore, the effects of lateral displacement on the axial strain estimation could be decoupled from those of the axial strain. The effects of axial strain on the axial displacement and strain estimation could be significantly reduced by using global axial stretching (Varghese and Ophir 1997a), a local, adaptive stretching method (Alam et al. 1998) or recorrelation techniques (Konofagou and Ophir 1998; Lee et al. 2007). In Kallel and Ophir (1997), the lateral displacements were assumed to be the same within the ultrasonic beams, because the lateral strain was small and the beams were relatively narrow. Therefore, the effects of lateral strain on the axial strain estimation were deemed negligible.

In this paper, we assume that the effects of axial displacement and strain on the lateral displacement estimation can be significantly reduced by using stretching or recorrelation techniques (Varghese and Ophir 1997a; Alam et al. 1998; Konofagou and Ophir 1998; Lee et al. 2007). The effects of lateral strain on the lateral displacement estimation are also deemed negligible.

In this paper, we study the effects of different parameters (i.e., pitch, beamwidth and interpolation method) on the lateral displacement estimation under well-controlled simulation and experimental conditions, which only consider lateral rigid motion. The axial displacement/strain and lateral strains are thus ignored to separate and emphasize the effects of lateral displacement. In the simulations, a homogeneous phantom is displaced in the lateral direction without any axial displacement/strain or lateral strain. Phantom experiments using lateral rigid motion are also performed to validate the simulation findings.

**METHODS**

**RF signal simulation**

The pre- and postdisplaced RF signals were generated using a 2-D convolution-based linear scattering model (Maurice and Bertrand 1999). The lateral component of a transducer PSF has a Gaussian shape whereas the axial component is a cosine function modulated by a Gaussian envelope (Maurice and Bertrand 1999). The axial component of a transducer PSF had a 60% –6 dB bandwidth and a 3.3-MHz center frequency, while the lateral PSF component had a full width at half maximum (FWHM) or a –6 dB beamwidth, varying between 1 and 6 mm. The original lateral pitch or pitch was equal to 0.0156 mm. The sampling frequency of the RF signals was 20 MHz. The speed of sound in soft tissues was assumed to be equal to 1540 m/s.

The scattering function consisted of point scatterers uniformly distributed in a rectangular sampling grid with a 100-mm width and a 50-mm depth. The grid intervals were equal to the original pitch (i.e., 0.0156 mm) in the lateral direction and the sampling interval (i.e., 0.0385 mm) in the axial direction. In the axial direction, the scatterer density was equal to 12 scatterers/wavelength, satisfying the requirement for fully developed speckle (Wagner et al. 1983). The strength, or echogeneity, of each scatterer followed a normal distribution.

The 2-D PSF was mapped onto the same sampling grid and was convolved with the scattering function to obtain the predisplaced RF signals. The scatterers were then moved in the lateral direction with an increment equal to the original pitch. In this way, the lateral displacement was always equal to integer multiples of the sampling interval so that the interpolation on the scattering function (Shapo et al. 1996) was not necessary. The post-displaced RF signals were then obtained from the convolution of the 2-D PSF and the postdisplaced scattering function.

Gaussian white noise was added to the simulated RF signals. The sonographic signal-to-noise ratio (SNR), defined as the root-mean-square (RMS) value of the previously simulated RF signal amplitude divided by the RMS of the additive noise amplitude, was set to be 60 dB, similar to what was considered in previous literature on tissue strain estimation (Walker and Trahey 1995) and myocardial elastography (Lee et al. 2007). Previous studies have also shown that the performance of myocardial elastography remains relatively unaffected when the SNR is equal to, or higher than 16 dB (Luo et al. 2008).

**Data processing in simulation**

In the lateral displacement estimation, the original RF signals were decimated (i.e., down-sampled) in the lateral direction. The decimation factor ranged from 10–80 to simulate a pitch from 0.156–1.25 mm. The simulated lateral displacement became a fraction of the pitch.

The pre- and postdisplaced RF signals were referred to as reference and comparison signals, respectively. The algorithm developed by Konofagou and Ophir (1998) was used to estimate the lateral displacements. The comparison RF signals were interpolated in the lateral direction, with an interpolation factor of 40. Unless otherwise stated, a linear interpolation method was used because it was the typical method used in literature (Konofagou and Ophir 1998).

The normalized cross-correlation function was used to obtain the lowest jitter error (Viola and Walker 2003).

\[1\] In the simulation described in this paper, we assume that for an unsteered linear array, the spacing between the centers of adjacent beams is equal to the spacing between adjacent array elements (i.e., pitch).
The window size was equal to 3.85 mm, with a 90% overlap. Because no axial motion was simulated, only a lateral 1-D search was performed to find the RF segment in the postinterpolated comparison signals that best matched the RF segment in the reference signals. The predefined search range was larger than the simulated lateral motion. Cosine interpolation was applied around the initial cross-correlation peak to obtain the sub-sample lateral displacements and improve the precision of estimation (Céspedes et al. 1995).

The average estimated lateral displacement was calculated in a region-of-interest (ROI) of 50 × 50 mm² located at the center of the simulated phantom. The ROI was smaller than the simulated phantom to reduce the boundary effects of the RF signal simulation and lateral displacement estimation. The bias of the estimation was obtained as the simulated lateral motion subtracted from the average estimates, whereas the jitter was calculated to be equal to the standard deviation (SD) of the estimates (Pinton and Trahey 2006). The mean and SD of the correlation coefficient were also calculated in the same ROI.

**Phantom preparation**

A polyacrylamide tissue-mimicking phantom was constructed, prepared in the following manner: a premixed 40% liquid acrylamide (19:1 acrylamide:bis-acrylamide ratio) (Thermo Fisher Scientific Inc., Waltham, MA, USA) was diluted in deionized water to produce an acrylamide concentration of 40% (weight/volume). The resulting solution was dissolved (1.75 mL per total mL) in 1M trishydroxymethylaminomethane (TRIS, 1.0 mL per total mL) with deionized water (7.16 mL per total mL). Ten percent (10%) ammonium persulfate (APS, 8.4 µL per total mL) and N,N,N',N′-tetramethylethylenediamine (TEMED, Sigma-Aldrich, St. Louis, MO, USA; 0.5 µL per total mL) were subsequently added. The mixture was allowed to polymerize at room temperature for approximately 15 min before use.

**Data acquisition**

As shown in Fig. 1, the phantom was placed in a water tank and subsequently immersed in degassed water. A linear-array transducer (model 10L5, Tera- son Ultrasound, Burlington, MA, USA) was attached to a computer-controlled positioner (Velmex Inc., Bloomfield, NY, USA) and placed below the water surface but without contact to the phantom. Efforts were made to align the lateral and axial directions of the transducer to the horizontal and vertical directions of the positioner, respectively. The 128-element linear array had a center frequency of 7 MHz. The pitch was approximately equal to 0.30 mm. A Terason 2000 ultrasound system (Teratech Corp., Burlington, MA, USA) was used to drive the transducer. The transmit focus was at a depth of 2.8 cm, whereas dynamic focusing was used in the receive aperture. The RF signals were acquired at a sampling frequency of 30 MHz and a frame rate of about 50 Hz. Each RF frame had 256 beams, which are twice the number of the elements because of the use of beamforming techniques.

The transducer was then moved by the positioner into the horizontal (or approximately, lateral) direction at a step of 0.015 mm and at a speed of 1 mm/s. Forty steps were performed to reach a maximum lateral motion of 0.6 mm. Five seconds after the transducer was moved to each position, the RF signals were acquired at a sampling frequency of 30 MHz and a frame rate of about 50 Hz. In the lateral displacement estimation, only one pair of RF frames was used for each step. Therefore, the RF frame rate or the speed at which the probe was moved, did not affect the results, because the transducer moved between acquisitions.

**Displacement estimation**

The reference used, with respect to which displacement was estimated, was the first RF frame acquired. The lateral motion between the comparison and reference signals ranged within 0–0.6 mm. The axial and lateral displacements were estimated using 1-D cross-correlation and recorrelation methods in a 2-D search (Lee et al. 2007). A normalized cross-correlation function was used with a window size of 2.57 mm and a 90% overlap. The RF signals in the comparison frame were laterally interpolated to obtain precise lateral displacement estimation (Konofagou and Ophir 1998; Lee et al. 2007). The interpolation factor was equal to 40. A linear interpolation method was used, unless otherwise stated. 2-D cosine interpolation was applied around the initial cross-correlation peak to obtain the subsample 2-D displacement (Céspedes et al. 1995; Luo and Konofagou 2009).

To study the effects of the pitch on lateral displacement estimation, the acquired RF signals were decimated by a factor of 4 or 2 (i.e., from 256 to 128, or 64, beams) in the lateral direction. Whether 256 or 128 beams were used, the pitch remained the same, i.e., 0.3 mm. When
64 beams were used, the pitch was assumed to increase to 0.6 mm. To compare the performance of different beamwidths, three ROIs were selected, one near the focal zone and the other two 6.5 mm away from the focal zone. The centers of the ROIs were at depths of 2.15, 2.8 and 3.45 cm, respectively. The ROI size was equal to 5 × 5 mm².

**Experimental beamwidth measurement**

The beamwidth of the linear array was measured at the three regions mentioned before. A 5-0 (1.0 metric) braided thread (Ethicon Inc., Somerville, NJ, USA) was immersed in degassed water and fixed to line along the elevational direction of the linear array. The linear array was moved by the positioner in the axial and lateral directions, until the thread was placed and imaged at the center of the previously chosen ROI. The RF signals were then acquired and the beamwidth was measured at the center of each ROI as the width (FWHM) of the backscattered signals from the thread, i.e., the FWHM of the beam profile along the lateral direction, assuming that the beamwidth within each ROI was the same. For ROIs I, II and III, the beamwidths were measured to be equal to 0.9, 1.3 and 1.7 mm, respectively.

**RESULTS**

**Simulation results**

The simulation results are organized as follows. Figure 2 shows the effects of the magnitude of the lateral displacement on lateral displacement estimation at a fixed pitch (0.625 mm) and beamwidth (2 mm). Figure 3 compares the estimator performance at different pitches at a fixed beamwidth (2 mm). Figure 4 depicts the beamwidth effects on the lateral displacement estimation at a fixed pitch (0.625 mm). Figure 5 illustrates the performance of the lateral displacement estimation at the same beam overlap (1—pitch/beamwidth (Konofagou and Ophir 1998)) of 68.75%. Figure 6 compares the linear and cubic spline methods in the RF signal interpolation in the lateral direction. In each figure, the estimated lateral displacement images are first compared. The performance indices (i.e., bias, jitter and correlation coefficient) at different parameters (i.e., pitch, beamwidth, etc.) are then plotted against the lateral displacement.

At the same pitch and beamwidth, the estimation performs best when the displacement is equal to 0 or the pitch (Fig. 2a and 2e). Figure 2c appears to be the noisiest, followed by Fig. 2b and 2d. From Fig. 2a–2e, the performance of the lateral displacement estimation deteriorates first (Fig. 2a to 2c or, 0 to 0.5 pitch) and then improves (Fig. 2c to 2e or, 0.5 to 1 pitch). Figure 2f shows that the estimation algorithm performs reliably over the entire range of the lateral displacement (i.e., 0 to 3 pitches). A linear relationship between the true and estimated displacements is shown (r > 0.999).

As shown in Fig. 2g, 2h and 2i, all three performance indices undergo a periodic variation, with the period equal to the pitch. When the displacement is equal to half-integer pitch multiples (e.g., 0, 0.5, 1, 1.5 or 2 pitches), the bias is the lowest and nearly zero. The bias is maximum when the displacement is approximately ± 0.2 pitch away from the beams. The jitter reaches the minimum at integer pitch multiples (e.g., 0, 1 or 2 pitches) and maximum at odd half-integer pitch multiples (e.g., 0.5, 1.5 or 2.5 pitches), respectively. On the other hand, the correlation coefficient is maximum at integer pitch multiples and minimum at odd half-integer pitch multiples, respectively.

Figure 2j shows the signal-to-noise ratio (SNR) of the lateral displacement estimation as a function of the lateral displacement. The SNR is defined as the ratio of the average over the SD (i.e., jitter) of estimation. As shown in Fig. 2j, in addition to the oscillation because of the periodic variation of the jitter, the SNR profile also shows an increased trend with the lateral displacement, suggesting that a larger lateral displacement is preferred to obtain a higher SNR.

As shown in Fig. 3a–3c, at the same beamwidth of 2 mm, the estimation noise drops as the pitch becomes smaller. The bias, jitter and correlation coefficient variation with the lateral displacement in Figs. 3e–3g have the same periodic patterns as in Fig. 2 (e.g., with a period equal to the pitch). More importantly, the estimator performance improves as the pitch decreases, i.e., the jitter and bias drop while the correlation coefficient increases. The 3-D plot of the jitter against the pitch and lateral displacement in Fig. 3d shows the same variation as Fig. 3f.

Figure 4a–4c shows that larger beamwidth reduces the estimation noise. As evident in Fig. 4e–4g, both the bias and jitter decrease while the correlation coefficient increases when the beamwidth increases. The 3-D plot of the jitter against the beamwidth and lateral displacement in Fig. 4d also shows lower jitter at larger beamwidths.

If the beam overlap is fixed, the estimation noise increases with the pitch (Fig. 5a to 5c). The correlation coefficient remains the same, regardless of the pitch (Fig. 5h). The bias and jitter increase with the pitch (Fig. 5d and 5e). At a fixed beam overlap, a larger pitch also denotes a larger beamwidth. These two parameter changes have opposite effects, as discussed and shown in Figs. 3 and 4. Results in Fig. 5d and 5e indicate that the effects of the pitch on the lateral displacement estimation are significantly larger than those of the beamwidth. On the other hand, when the bias and jitter are normalized by the pitch and plotted against the lateral displacement, also normalized by the pitch, they become relatively independent of the pitch (or the beamwidth).
Fig. 5f and 5g. Results in Fig. 5d and 5e can be explained by the fact that the lateral displacement estimation is first measured in subpitch multiples and then converted into millimeters by multiplying by the pitch.

In Fig. 6a and 6b, the spline interpolation on the RF signals is found to obtain significantly smoother displacement estimates, as confirmed by the lower jitter and higher correlation coefficient (Fig. 6d and 6e). The trade-off is that the bias of the spline interpolation is higher, but still on the same order of 0.1 μm as the other method (Fig. 6c). As shown in Fig. 6f, the subpitch, or interpolated, RF signals using the spline interpolation reconstruct the original signals significantly better than those using the linear interpolation. This may explain...
the decreased jitter associated with the spline compared to that with the linear interpolation (Fig. 6d). If the jitter noise is more important than the bias, which may be true in most applications, the spline interpolation would be advisable.

**Phantom results**

At a total beam number of 128 and a pitch of 0.3 mm, the estimation performance deteriorates for lateral displacements between 0 and 0.5 pitch multiples (Fig. 7b to 7d), whereas it improves between 0.5 and 1 pitch multiples (Fig. 7d to 7f), *i.e.*, in agreement with the simulation results (Fig. 2a to 2c).

When 256, or 128, beams are used (*i.e.*, pitch = 0.3 mm), a linear relationship between the true and estimated lateral displacements is shown in Fig. 7g (*r > 0.999*). By using 64 beams (*i.e.*, pitch = 0.6 mm), however, the displacement is underestimated, or overestimated, when the displacement is between 0 and 0.3 mm, or between 0.3 and 0.6 mm, respectively, because of the presence of estimation bias.

Using a total beam number of 64, the bias is minimum when the displacement is 0, 0.5 or 1 pitch multiples (*i.e.*, 0, 0.3 or 0.6 mm) (Fig. 7h), *i.e.*, when the interpolation or RF signals is not required or adjacent beams contribute to the interpolated, sub-beams equally (Appendix A). The bias for the 0.30-mm pitch (*i.e.*, 256

---

Fig. 3. The estimated lateral displacement images when the applied lateral displacement is 0.3125 mm and the pitch is (a) 1.25 mm, (b) 0.625 and (c) 0.3125 mm, respectively, and (d) 3-D plot of the estimation jitter varied with the pitch and lateral displacement, (e) the bias and (f) jitter of the estimation and (g) correlation coefficient (average ± SD) as a function of the lateral displacement at different pitch. (beamwidth = 2 mm)
or 128 beams) is very small (< 0.005 mm) and does not clearly exhibit a periodic variation with the lateral displacement. For both 0.30- and 0.60-mm pitches, the jitter is the lowest (or the highest), and the correlation coefficient is the highest (or the lowest), when the displacement is equal to integer pitch multiples (or at odd half-integer pitch multiples) (Fig. 7i). As is evident in Fig. 7h, 7i, and 7j, a pitch of 0.3 mm could result in significantly lower bias and jitter and higher correlation coefficient, demonstrating the importance of a smaller pitch in the lateral displacement estimation. The maximum jitter is approximately equal to 10 μm when a pitch of 0.3 mm is used. The maximum jitter of the lateral displacement estimation is on the same order of the simulation values found by Walker and Trahey (1994).

Figure 7j also shows that the correlation coefficient is slightly higher at increased beam density from 128–256 beams. When 256 beams are used, the jitter is also slightly lower, except at lateral displacements around 0.15 or 0.45 mm (Fig. 7i). The bias remains similar in both cases (Fig. 7h). The variation period is also the same in both cases because the pitch is the same (0.3 mm). The nonsignificant difference between the use of 256 and 128 beams suggests that the fundamental limit of the lateral displacement estimation may thus be mainly determined by the original pitch in the linear array, and the lateral estimator performance is not expected to be significantly improved with an increased beam density by using beamforming techniques.

Figure 8 compares the performance of the estimation in ROIs I, II, and III, as indicated in Fig. 7a. The same
periodic variation can be observed in the jitter and correlation coefficient of three ROIs. In addition, the jitter is the lowest in region III and highest in region I, except for displacements between 0.5 and 0.6 mm, whereas the correlation coefficient is the highest in region III and the lowest in region I. These results can be explained by the largest beamwidth in region III (1.7 mm) and smallest beamwidth in region I (0.9 mm). Although similar across the three ROIs, the bias appears slightly lower in region III than in regions I and II (Fig. 8a), because of the wider beamwidth, similar to the simulation results (Fig. 4). These experimental results in different regions with different beamwidths are thus in good agreement with the simulation results (Fig. 4).
Both the pitch and beamwidth affect the performance of the lateral displacement estimation. A smaller pitch or a larger beamwidth is preferred, because both contribute to a higher beam overlap (Konofagou and Ophir 1998). However, simulation results also confirm that the pitch effects on the lateral estimation performance are larger than the beamwidth effects. For example, the jitter difference between the pitch of 1.25 and 0.625 mm is four-fold (Fig. 3f), whereas the jitter difference between the beamwidth of 1 and 2 mm is twice (Fig. 4f). The phantom experimental findings are consistent with the simulation results. Greater effects of the pitch are shown in Figs. 7 and 8. The jitter increases by a six-fold when the pitch increases from 0.3–0.6 mm (Fig. 7i), but less than twice when the beamwidth decreases from 1.7 to 0.9 mm (Fig. 8b).

Finally, in Fig. 9a–9c, the spline interpolation is found to have lower jitter and higher correlation coefficient and bias, which is consistent with the simulations findings (Fig. 6). The maximum jitter is reduced from 10 to 5 μm when the spline instead of linear interpolation is used.

**DISCUSSION**

Most biological soft tissues are nearly incompressible (Fung 1993); the lateral and elevational tissue strains are approximately equal to half of the axial strain. Therefore, lateral motion of tissues is inevitable in elastography. In
this paper, a lateral rigid motion was therefore assumed in simulation to evaluate the performance of a lateral displacement estimator. In the phantom experiments, the phantom did not undergo any deformation while the transducer was moved in the lateral direction. In both simulation and phantom results, the correlation coefficients between the pre- and postdisplaced RF signals were much higher than in the presence of strains, because only lateral rigid motion was considered.

The results of the simulation and phantom experiments clearly demonstrate a periodic variation in the performance of the lateral displacement estimation (i.e.,...
bias, jitter and correlation coefficient) at different lateral displacements, with the period equal to the pitch. The reader is referred to Appendix A for a brief explanation regarding the periodic variation. From the periodic variation of the displacement estimation performance, expected consequences on the strain estimation can be elucidated (Appendix B). The performance analysis of the lateral strain estimation (Konofagou et al. 2000b) is beyond the scope of this work, because only lateral rigid motion is considered. Improved performance in the lateral displacement estimation is expected to result in higher quality in the lateral strain images. Associated effects on the lateral strain estimation (e.g., periodic variation in the strain estimation performance and the “zebra” artifacts) are discussed in Appendix B.

As shown in Figs. 3 and 4, to reduce the bias and jitter of the lateral displacement estimation, a smaller pitch and a larger beamwidth are preferred. At smaller pitches, the transducer provides more beams, which are the reliable data instead of the postinterpolation, reconstructed beams. However, small pitch may complicate the design of the transducer. At larger beamwidths, the adjacent beams share more ultrasound scatterers and the RF signals of these beams are more statistically dependent. Therefore, it may be more accurate to interpolate the RF signals between adjacent beams. As a result, increasing the beamwidth reduces the jitter and bias. However, the trade-off is the reduced lateral resolution (Righetti et al. 2003) and will thus be dictated by the specific application.

As discussed in Appendix A, the performance of the lateral displacement estimation relies on the accuracy of the lateral interpolation of RF signals. If sinc interpolation is used, the subpitch noiseless signals can be in theory fully reconstructed, as long as the sampling frequency is higher than the Nyquist frequency. Therefore, the correlation coefficient between the reference RF signals and the comparison, interpolated RF signals is equal to 1, in the presence of lateral displacement only. However, the sinc interpolation theoretically needs an infinite signal duration; otherwise, it tends to produce ripple artifacts, i.e., the ringing effect. Although a weighting function is suggested for sinc interpolation of a finite-length signal, the optimal shape and length of the weighting function may vary across applications. Given the small number of beams (e.g., 64 to 256) on clinical ultrasound images, the sinc interpolation may not be considered practical for RF signal interpolation in the lateral direction. Therefore, linear interpolation is used widely in lateral displacement estimation in elastography because it is simple and straightforward (Konofagou and Ophir 1998; Lee et al. 2007). Results in Figs. 6 and 9 demonstrate that the cubic spline interpolation can obtain lower jitter and higher
correlation coefficient, at the cost of slightly increased bias. This can be explained by the fact that the cubic spline interpolation may provide more accurate subpitch signals between beams, because the data are interpolated via a piecewise polynomial function that preserves second-derivative continuity (Fig. 6f). One possible reason for the increased bias of the spline interpolation is that the spline interpolation preserves second-derivative continuity and thus is more sensitive to the variation of the pre-interpolated signals. However, this remains to be further investigated. The cubic spline interpolation may be preferred over the linear interpolation in elastography because the bias is typically much lower than jitter. In addition, in the phantom experiments, the difference in the bias between the linear and spline interpolation methods was not as significant (Fig. 9(a)) as in the simulations (Fig. 6(c)), while decreased jitter and increased correlation coefficient using spline interpolation are equally evident (Fig. 9(b) and Fig. 9(c)).

In myocardial elastography, where a phased-array transducer is typically used, the beam spacing increases with depth and is not equal to the pitch (Luo et al. 2008). According to the findings in this study, a shallow depth is expected to obtain lower bias and jitter than a deeper region in a phased array configuration because the beam spacing is smaller. However, the fundamental limit of the lateral displacement estimation at a specific depth may still be determined by the pitch, even though some beamforming schemes are used to increase the beam number, similar to the results shown in Fig. 7g–7j. In addition, both the beamwidth and displacement vary spatially. The combined effects of all these factors on the lateral displacement estimation in myocardial elastography using a phased array will be explored in future studies.

The results of the lateral displacement estimation in this study can also be extended into the elevational displacement estimation because the same estimation methods in 3-D are used (Konofagou and Ophir 2000; Lee and Konofagou 2008) and other estimation algorithms such as sum absolute difference (SAD), sum squared difference (SSD) and methods using 2-D kernels Similarly, smaller pitch and larger beamwidth in the elevational directions are expected to improve the performance of the elevational displacement estimation in terms of bias, jitter and correlation coefficient.

CONCLUSION

Simulations and phantom experiments were performed to investigate the key parameters on the lateral displacement estimation in ultrasound elastography. A lateral rigid motion configuration was applied to separate
it from the effects of axial displacement/strain and lateral strain on lateral displacement estimation. The estimator performance as indicated by the bias, jitter and correlation coefficient shows a periodic variation with lateral displacement, with the period equal to the pitch. Because of the periodic variation, a larger lateral displacement might be preferred to improve the SNR of the estimation. The performance was found to improve with a decreased pitch and/or an increased beamwidth. A smaller pitch was preferred at the same beam overlap, because the effects of pitch on the lateral displacement estimation appeared to be more significant than those of the beamwidth. The cubic spline interpolation of the RF signals in the lateral direction decreased the jitter but increased the bias. In summary, smaller pitch, wider beamwidth and spline interpolation were concluded to be optimal in reducing the jitter error of the lateral displacement estimation. A pitch of 0.3 mm (or, a beam density of 128 beams within 3.8 cm) and a beamwidth of 1.7 mm were found to be sufficient for precise lateral displacement estimation with the jitter less than 10 and 5 microns using linear and spline interpolation, respectively.

Acknowledgements—This study was supported in part by the American Heart Association (SDG0353444T), National Institutes of Health (R01EB006042) and the Wallace H. Coulter foundation. The authors appreciate Caroline Maleke, M.S., and Yao-Sheng Tung, M.S., in our laboratory for preparing the phantom used in this study. The authors also wish to thank Wei-Ning Lee, M.S., and Jean Provost, M.S., in our laboratory for all helpful discussions.

REFERENCES


interpolated beam $B_i$. A single estimation of the lateral displacement may not be required while the correlation coefficient between $A_0$ and $B_0$ and the adjacent beam $B_1$.

Assume the predeformation RF signal in the reference frame is $A_0$ and the corresponding RF signal at the same beam location in the comparison frame is $B_0$. The lateral displacement is assumed to be between 0 and 1 pitch; therefore, the most matched subpitch RF beam $B_i$ is between beam $B_0$ and the adjacent beam $B_1$.

When the displacement is equal to 0, the RF lateral interpolation may not be required while the correlation coefficient between $A_0$ and $B_0$ reaches the maximum value of 1. The interpolated peak is expected to be very close to the pre-interpolated peak. Therefore, the bias and jitter would be the lowest.

When the lateral displacement increases but is smaller than one half of the pitch, the contribution of beam $B_0$ to the interpolated beam $B_i$ decreases. However, when the lateral displacement is larger than half a pitch, beam $B_0$ contributes to the interpolated beam $B_i$ more than $B_1$ does. Because the bias can be either positive or negative while the jitter is always positive, the bias and jitter as a function of the lateral displacement is antisymmetric or symmetric, respectively, at half a pitch. When the displacement is equal to half a pitch, the estimation becomes unbiased probably because beams $B_0$ and $B_1$ contribute equally or similarly to the interpolated beam $B_i$. A single estimation of the lateral displacement can be either underestimated or overestimated. The averaged error, or bias, is nearly 0. However, the standard deviation, or jitter, reaches a maximum value.

When the displacement ranges from 1–2 pitch multiples, or from 2–3 pitch multiples, etc., the same variation repeats. Therefore, the performance of the lateral displacement estimation shows a periodic variation, with the period equal to the pitch. The findings are very similar to previous results on the bias and/or jitter of time delay estimation (Céspedes et al. 1995, Hoyt et al. 2006). In those studies, the period of the bias and/or bias is equal to the sampling interval of the echo signals. The bias is caused through the curve fitting applied to the cross-correlation function (Céspedes et al. 1995). The period of the bias and jitter in lateral displacement estimation in this paper is equal to the pitch (i.e., the sampling interval in the lateral direction). The bias is a result of the lateral interpolation of the RF signals as well as the curve fitting applied to the cross-correlation function. The effects of the former source of bias are usually larger than the latter when the interpolation factor is adequate.

If the beam overlap and the lateral displacement in subpitches (i.e., lateral displacement normalized by the pitch) are both fixed, the reference beam $A_0$ and comparison beam $B_0$ share the same percentage of scatterers regardless of the pitch, because both the beamwidth and the lateral displacement in millimeters vary simultaneously. The correlation between $A_0$ and $B_0$ is therefore the same. Similarly, the correlation between beam $A_0$ and $B_1$ does not vary with the pitch. After the interpolation applied to the RF signals to obtain the sub-beam signals between $B_0$ and $B_1$, it is intuitive to assume that the correlation function follows the same shape in the lateral direction, and the width of the correlation function in subpitch samples does not depend on the pitch. Therefore, the correlation coefficient, as well as the bias and jitter of the lateral displacement estimation, both measured in sub-pitches, is relatively independent of the pitch, as shown in Fig. 5f–5h. However, similar bias and jitter measured in subpitches in Fig. 5f and 5g are converted to larger values at larger pitches, when measured in millimeters (Fig. 5d and 5e).

APPENDIX A

EXPLANATION OF THE PERIODIC VARIATION OF THE PERFORMANCE

The lateral displacement estimation relies on RF signal interpolation in the lateral direction, which produces subpitch information. Assume the predeformation RF signal in the reference frame is $A_0$ and the corresponding RF signal at the same beam location in the comparison frame is $B_0$. The lateral displacement is assumed to be between 0 and 1 pitch; therefore, the most matched subpitch RF beam $B_i$ is between beam $B_0$ and the adjacent beam $B_1$.

When the displacement is equal to 0, the RF lateral interpolation may not be required while the correlation coefficient between $A_0$ and $B_0$ reaches the maximum value of 1. The interpolated peak is expected to be very close to the pre-interpolated peak. Therefore, the bias and jitter would be the lowest.

When the lateral displacement increases but is smaller than one half of the pitch, the contribution of beam $B_0$ to the interpolated beam $B_i$ decreases. However, when the lateral displacement is larger than half a pitch, beam $B_0$ contributes to the interpolated beam $B_i$ more than $B_1$ does. Because the bias can be either positive or negative while the jitter is always positive, the bias and jitter as a function of the lateral displacement is antisymmetric or symmetric, respectively, at half a pitch. When the displacement is equal to half a pitch, the estimation becomes unbiased probably because beams $B_0$ and $B_1$ contribute equally or similarly to the interpolated beam $B_i$. A single estimation of the lateral displacement can be either underestimated or overestimated. The averaged error, or bias, is nearly 0. However, the standard deviation, or jitter, reaches a maximum value.

When the displacement ranges from 1–2 pitch multiples, or from 2–3 pitch multiples, etc., the same variation repeats. Therefore, the performance of the lateral displacement estimation shows a periodic variation, with the period equal to the pitch. The findings are very similar to previous results on the bias and/or jitter of time delay estimation (Céspedes et al. 1995, Hoyt et al. 2006). In those studies, the period of the bias and/or bias is equal to the sampling interval of the echo signals. The bias is caused through the curve fitting applied to the cross-correlation function (Céspedes et al. 1995). The period of the bias and jitter in lateral displacement estimation in this paper is equal to the pitch (i.e., the sampling interval in the lateral direction). The bias is a result of the lateral interpolation of the RF signals as well as the curve fitting applied to the cross-correlation function. The effects of the former source of bias are usually larger than the latter when the interpolation factor is adequate.

If the beam overlap and the lateral displacement in subpitches (i.e., lateral displacement normalized by the pitch) are both fixed, the reference beam $A_0$ and comparison beam $B_0$ share the same percentage of scatterers regardless of the pitch, because both the beamwidth and the lateral displacement in millimeters vary simultaneously. The correlation between $A_0$ and $B_0$ is therefore the same. Similarly, the correlation between beam $A_0$ and $B_1$ does not vary with the pitch. After the interpolation applied to the RF signals to obtain the sub-beam signals between $B_0$ and $B_1$, it is intuitive to assume that the correlation function follows the same shape in the lateral direction, and the width of the correlation function in subpitch samples does not depend on the pitch. Therefore, the correlation coefficient, as well as the bias and jitter of the lateral displacement estimation, both measured in sub-pitches, is relatively independent of the pitch, as shown in Fig. 5f–5h. However, similar bias and jitter measured in subpitches in Fig. 5f and 5g are converted to larger values at larger pitches, when measured in millimeters (Fig. 5d and 5e).

APPENDIX B

EXPECTED CONSEQUENCES OF THE PERIODIC VARIATION OF THE PERFORMANCE

Assume the lateral displacement $u$ of a homogeneous phantom undergoing a uniform lateral strain $\epsilon_x$ is given by

$$u = \epsilon_x x,$$

where $x$ is the beam position relative to the axis, with zero lateral displacement.

If the lateral strain $x$ is fixed, the lateral displacement $u$ is proportional to the beam position $x$. According to the periodic variation of the lateral displacement estimation performance with the lateral displacement as shown in Fig. 2, the performance of the lateral displacement estimation also periodically varies with the beam position. The period is equal to pitch $h$. At a fixed beam position $x$, the bias and jitter of the lateral displacement estimation as a function of the lateral strain is also expected to vary periodically, with a period equal to pitch $h$.

When the lateral strains vary spatially, the lateral displacement is expected to have a similar variation with the beam position. However, the period depends on the local lateral strain distribution. It has been shown that the periodic variation of the bias in the axial displacement estimation may induce periodic bright and dark artifact in the axial strain images, i.e., "zebra" (Céspedes et al. 1995; Ophir et al. 1996). Similarly, the periodic bias of lateral displacement estimation may also result in the "zebra" artifact in the lateral strain images.

The performance of the lateral displacement estimation is mainly determined by the jitter because the bias is at least one order of magnitude lower. Because of the periodic variation as shown in Fig. 2, the jitter remains the same when the lateral motion increases by integer pitch multiples. In this case, the SNR of the lateral displacement estimation increases with the lateral displacement. Figure 2j suggests that a larger lateral displacement is preferred to obtain a higher SNR.

In the presence of lateral strain, similar results could be expected. A relatively larger lateral strain might obtain a higher SNR. However, in elastography, a larger strain usually means larger axial strain, due to the underlying tissue incompressibility which can deteriorate the correlation between the pre- and postdeformed signals. Therefore, there exists a trade-off, which has been shown in the strain filter framework for the lateral strain estimation (Konofagou et al. 2000b).